ABSTRACT

An exploratory investigation designed to gain insights into children's mathematical formulation of observed actions upon objects is presented. Eight episodes in which first and second graders were asked to interpret, in terms of number sentences, a sequence of actions with unifix cubes are also presented. Results of analysis of the videotaped episodes are presented and discussed in relation to children's concepts of equality. (MS)

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First and Second Grade Children's Interpretation of Actions upon Objects

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PREFACE

This publication shares with the interested individuals the results of an exploratory investigation designed to gain insights into the children's mathematical formulation of observed actions upon objects. It is hoped that the reader interested in research on young children's mathematical thinking will find this publication a source of ideas for further exploration of this area.

A special gratitude is expressed to two doctoral students in Mathematics Education at the Florida State University: Patricia Campbell, for assisting the author with the managerial aspects of the interviews, and Max Gerling, for videotaping the interviews.

Thanks are due to the Project administrative assistant, Janelle Hardy, for coordinating the technical aspects of the preparation of the report, and to Joe Schmerler for the typing.

FOREWORD

Ed Begle recently remarked that curricular efforts during the 1960's taught us a great deal about how to teach better mathematics, but very little about how to teach mathematics better. The mathematician will, quite likely, agree with both parts of this statement. The layman, the parent, and the elementary school teacher, however, question the thesis that the "new math" was really better than the "old math." At best, the fruits of the mathematics curriculum "revolution" were not sweet. Many judge them to be bitter.

While some viewed the curricular changes of the 1960's to be "revolutionary," others disagreed. Thomas C. O'Brien of Southern Illinois University at Edwardsville recently wrote, "We have not made any fundamental change in school mathematics."[1] He cites Allendoerfer who suggested that a curriculum which heeds the ways in which young children learn mathematics is needed. Such a curriculum would be based on the understanding of children's thinking and learning. It is one thing, however, to recognize that a conceptual model for mathematics curriculum is sound and necessary and to ask that the child's thinking and learning processes be heeded; it is quite another to translate these ideas into a curriculum which can be used effectively by the ordinary elementary school teacher working in the ordinary elementary school classroom.

Moreover, to propose that children's thinking processes should serve as a basis for curriculum development is to presuppose that curriculum makers agree on what these processes are. Such is not the case, but even if it were, curriculum makers do not agree on the implications which the understanding of these thinking processes would have for curriculum development.

In the real world of today's elementary school classroom, where not much hope for drastic changes for the better can be foreseen, it appears that in order to build a realistic, yet sound basis for the mathematics curriculum, children's mathematical thinking must be studied intensively in their usual school habitat. Given an opportunity to think freely, children clearly display certain patterns of thought as they deal with ordinary mathematical situations encountered daily in their classroom. A videotaped record of the outward manifestations of a child's thinking, uninfluenced by any teaching on the part of the interviewer, provides a rich source for conjectures as to what this thinking is, what mental structures the child has developed, and how the child uses these structures when dealing with the ordinary concepts of arithmetic. In addition, an intensive analysis of this videotape generates some conjectures as to the possible sources of what adults view as children's "misconceptions" and about how the school environment (the teacher and the materials) "fights" the child's natural thought processes.

The Project for the Mathematical Development of Children (PMDC)[2] set out

[1] "Why Teach Mathematics?" The Elementary School Journal 73 (Feb. 1973), 258-268.

[2] PMDC is supported by the National Science Foundation, Grant No. PES 74-18103-A03.

to create a more extensive and reliable basis on which to build mathematics curriculum. Accordingly, the emphasis in the first phase is to try to understand the children's intellectual pursuits, specifically their attempts to acquire some basic mathematical skills and concepts.

The PMDC, in its initial phase, works with children in grades 1 and 2. These grades seem to comprise the crucial years for the development of bases for the future learning of mathematics, since key mathematical concepts begin to form at these grade levels. The children's mathematical development is studied by means of:

1. One-to-one videotaped interviews subsequently analyzed by various individuals.

2. Teaching experiments in which specific variables are observed in a group teaching setting with five to fourteen children.

3. Intensive observations of children in their regular classroom setting.

4. Studies designed to investigate intensively the effect of a particular variable or medium on communicating mathematics to young children.

5. Formal testing, both group and one-to-one, designed to provide further insights into young children's mathematical knowledge.

The PMDC staff and the Advisory Board wish to report the Project's activities and findings to all who are interested in mathematical education. One means for accomplishing this is the PMDC publication program.

Many individuals contributed to the activities of PMDC. Its Advisory Board members are: Edward Begle, Edgar Edwards, Walter Dick, Renee Henry, John LeBlanc, Gerald Rising, Charles Smock, Stephen Willoughby, and Lauren Woodby. The principal investigators are: Merlyn Behr, Tom Denmark, Stanley Erlwanger, Janice Flake, Larry Hatfield, William McKillip, Eugene D. Nichols, Leonard Pikaart, Leslie Steffe, and the Evaluator, Ray Carry. A special recognition for this publication is given to the PMDC Publications Committee consisting of Merlyn Behr (Chairman), Thomas Cooney, and Tom Denmark.

Eugene D. Nichols,
Director of PMDC

THE EXPERIMENT

As part of several types of research activities of the Project for the Mathematical Development of Children, a clinical study of first and second grade children was carried out at an elementary school of about 1,000 children in the southeast. The purpose of the study was to find out how children interpret, in terms of number sentences, certain actions performed on physical objects. The objects used were single unifix cubes. To obtain uniformity of stimuli, a sequence of actions on the cubes was recorded on a videotape. The author performed the actions upon the cubes and subsequently used the tape individually with children.

The sequence of events in interviewing each child individually was as follows:

Step 1. After the child wrote his/her name on a sheet of paper, the experimenter said:

> How about writing a number sentence for me--any number sentence you like?

If the child wrote something that was not considered a number sentence (examples appear later in the text), the experimenter said:

> How about now writing something that has a plus or a minus and an equals sign?

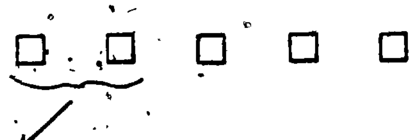
Step 2. Next the experimenter said:

> Now I am going to talk to you on TV. I'll tell you to do something. You watch and do it, OK?

Step 3. The eight action episodes were shown to the child on a 20-inch screen. After each episode, the tape was stopped, the child wrote a sentence, and then the next episode was shown.

Each of the eight episodes was presented in the same mode. The first episode is fully described below along with the instructions in the order in which they were presented. These instructions were also repeated in each episode.

Episode 1. Five unifix cubes are placed on a table as follows:

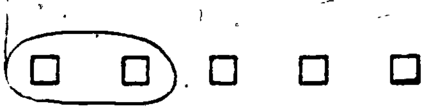


The experimenter points to each cube in silence, giving the child an opportunity to count the cubes. Then he says, "Watch carefully." Two blocks on the left (child's view) are pushed off the table (a strip of cardboard is used to assure that the blocks fall off simultaneously). Then the experimenter says, "Write a number sentence that tells what I did." The resulting configuration, after the blocks have been pushed off, remains visible on the screen for from three to five seconds, then is phased out. The child writes a sentence and is asked to read it. Then the next episode is presented in the same sequence.

Episode 2.

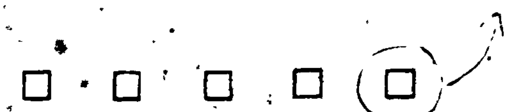
These three blocks are dropped from the table simultaneously.

Episode 3.



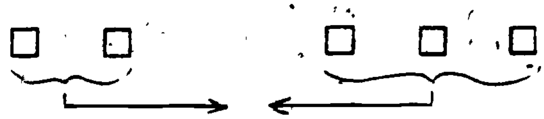
The experimenter picks up the two blocks with the right hand and removes then from the child's view.

Episode 4.



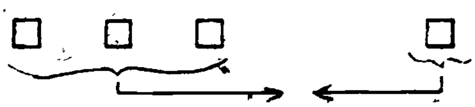
The experimenter picks up the one block with the left hand and removes it from the view of the child.

Episode 5.



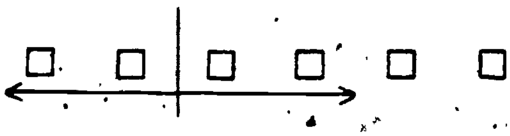
The experimenter pushes simultaneously the two and the three blocks together (two strips of cardboard are used for this purpose), so that one pile of blocks is formed.

Episode 6.



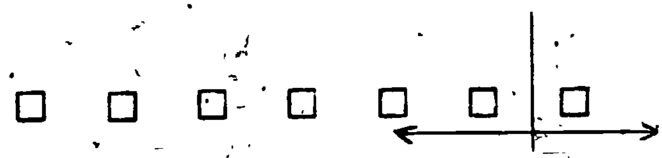
The experimenter pushes the three blocks and the one block together as in Episode 5.

Episode 7.



The experimenter, using two strips of cardboard, simultaneously pushes apart the two and the four blocks, so that two sets of blocks are obtained at the opposite ends of the table.

Episode 8.



The experimenter pushes the six and the one blocks apart, as in Episode 7.

THE RESULTS

As previously mentioned, it was necessary to ascertain the children had some referent for the phrase number sentence, thus the directions,

> How about writing a number sentence for me--any number sentence you like?

was given first. In response to these directions, the following are some examples of what children wrote.

First graders	Second graders
1 2 3 4 5 6 7 8 9 10 11 12	I like 2
11	I am nine years old
1 + 2 + 4 - 5	I had 5 pieces
	A boy is big
	1 2 3 4 5 6 7 8 9 10 11 12 13
	
	I am 6 years old
	5

It is interesting to note that the responses the first graders wrote to the request for a number sentence fall into these categories:

(1) a sequence of numbers, or

(2) a single number, or

(3) a phrase containing addition and subtraction.

In the examples above it can be seen that second graders are more flexible in interpreting a "number sentence." This interpretation embraces English sentences which refer to numbers as well as size.

Following the second set of instructions, How about now writing something that has a plus or a minus and an equal sign?

all children wrote number sentences.

The following is a summary of the results for 22 first graders (beginning of March) and 25 second graders (middle of October).

Episode 1. Five blocks on the table, two pushed off

Sentences written by children	Frequency: First graders	Frequency: Second graders
5 - 2 = 3	12 (6 horizontal, 6 vertical)	14 (12h, 2v)
5 - 3 = 2	0	3 (2h, 1v)
~~6 - 3 = 3~~	2 (h)	1 (h)
3	1	2
other	7	5

Episode 2. Five blocks on the table, three pushed off

Sentences written by children	First graders	Second graders
5 - 3 = 2	12 (6h, 6v)	21 (17h, 4v)
2	1	2
5 - 2 = 3	1 (v)	1 (h)
other	8	1

Episode 3. Five blocks on the table, two picked up

Sentences written by children	First graders	Second graders
5 - 2 = 3	12 (7h, 5v)	21 (17h, 4v)
3	1	2
other	9	2

Episode 4. Five blocks on the table, one picked up

Sentences written by children	First graders	Second graders
5 - 1 = 4	11 (6h, 5v)	19 (16h, 3v)
4	1	2
other	10	4

Episode 5. Two and three blocks pushed together

2 + 3 = 5	3 (2h, 1v)	9 (8h, 1v)
3 + 2 = 5	2 (h)	8 (7h, 1v)
5 - 0 = 5	4 (3h, 1v)	1 (h)
5	2	3
5 - 5 = 0	2 (1h, 1v)	0
other	9	4

Episode 6. Three blocks and one block pushed together

3 + 1 = 4	3 (2h, 1v)	12 (h)
4 - 0 = 4	4 (3h, 1v)	2 (1h, 1v)
4	2	2
1 + 3 = 4	1 (h)	1 (v)
other	12	8

Episode 7. Six blocks, four and two separated

6 - 2 = 4	9 (5h, 4v)	6 (h)
6 - 6 = 0	2 (1h, 1v)	4 (h)
6 - 0 = 6	3 (h)	2 (h)
6 - 4 = 2	0	2
6	0	2
0	2	0
other	6	9

Episode 8. Seven blocks, one and six separated

7 - 1 = 6	6 (4h, 2v)	11 (h)
7 - 6 = 1	1 (v)	4 (3h, 1v)
7 - 7 = 0	2 (1h, 1v)	2 (h)
7 - 0 = 7	0	2
7	2	0

0	2	0
other	9	6

DISCUSSION

In selecting the first six episodes the PMDC staff postulated "key" responses. They were as follows:

1. 5 - 2 = 3	4. 5 - 1 = 4
2. 5 - 3 = 2	5. 2 + 3 = 5 or 3 + 2 = 5
3. 5 - 2 = 3	6. 3 + 1 = 4 or 1 + 3 = 4

Accepting these as "correct responses," the percents of "success" are as follows:

Episode	First graders	Second graders
1	55%	56%
2	55%	84%
3	55%	84%
4	50%	76%
5	23%	68%
6	18%	52%

With the exception of the first episode, the second graders have given the expected response much more frequently than the first graders. It would probably be safe to ascribe this difference to the effect of the longer period of teaching, during which the predominant emphasis was on addition and subtraction.

It is interesting to note the differences in preferences for the horizontal over the vertical form of writing sentences. For the expected responses, the following are the percents of children who used the horizontal form (the "keyed" response is taken to be 100%).

Episode	First graders	Second graders
1	50%	48%
2	50%	81%
3	58%	81%
4	55%	84%

5	80%	88%
6	75%	93%

The second graders' greater preference for the horizontal form (except for Episode 1) can probably also be attributed to instruction; at that particular school the horizontal form was used more frequently than the vertical form.

The construction of Episodes 7 and 8 was motivated by the investigations of children's concept of equality, discussed in other PMDC publications[3]. The crucial observation made in those investigations was that first and second graders reject the equality form a = b + c as being "wrong" and "backward." The author a[illegible] construct a dynamic situation with manipulatives which mi[illegible]ggest to children this sentence form. The obvious manipulation seemed to be a motion separating simultaneously a set of objects into two subsets. From the following results, it is seen that the intended interpretation did not take place. It seems that the sentence form a + b = c or a - b = c is so strongly imbedded in children's thinking that they employ these forms to the exclusion of others in interpreting actions upon objects.

The following results were obtained for the last two episodes.

Episode 7. Six blocks, four and two separated

	First graders	Second graders
6 - 2 = 4	41%	24%
6 - 6 = 0	9%	16%
6 - 0 = 6	14%	7%
6 - 4 = 2	0%	12%
other	36%	40%

Episode 8. Seven blocks, one and six separated

7 - 1 = 6	27%	44%
7 - 7 = 0	9%	8%
7 - 0 = 7	14%	0%
7 - 6 = 1	5%	16%
other	45%	32%

[3] Behr, M., S. Erlwanger, and E. Nichols. How Children View Equality Sentences (PMDC Technical Report No. 3); and T. Denmark, E. Barco, and J. Voran. Final Report--A Teaching Experiment on Equality. Tallahassee, Florida: Florida State University, 1976.

The complete abstinence from writing the form a = b + c should be investigated further. Although children reject it as "wrong" and "backward," one might construct an experiment in which children could be enticed into pretending that a sentence like 6 = 4 + 2 is alright and then asked to tell a story about real objects which would fit this sentence. It would be important to search for models which seem sensible to children and which promote the concept of equality as an equivalence relation, rather than as an operator. A study carried out by Coleen Frazer[4] points out that even college students do not possess an operational concept of the symmetric property of equality. The ability of an individual to accept, with great ease, the symmetric and possibly other properties of equality, does not necessarily mean that this individual is able to work with equal success with the two symmetric forms.

This exploratory experiment suggested that children begin very early in their school days to formulate mental constructs about the very crucial concept of equality and this particular construct, possibly extremely inadequate, might persist throughout the later years.

Our informal observation of second graders whose teacher taught the children to use the phrase "is the same as" for the symbol "=" suggested that this phrase, rather than "is equal to" might be more conducive to children's mental construct of equality as a relation.

If one accepts the thesis that young children should indeed perceive mathematics as an "action" subject and that the primary goal should be to teach these children how to do mathematics and, furthermore, if one would want the symbolism to be isomorphic to students' thinking about the actions suggested by the symbols, then the conventional use of the equality symbol is inadequate. More than that, this use is contrary to children's perceptions. The symbol, which would be consistent with children's perception of mathematical operations would have to be a non-symmetric, one-way symbol. For example, the symbol $\longrightarrow$ in $(4 + 3) \longrightarrow 7$ would more closely correspond to how first and second graders think about addition. It would suggest that adding 4 and 3 results in 7. The same symbols in $7 \longrightarrow (4 + 3)$ should then possibly suggest separating 7 into 4 and 3. The latter situation, however, raises the question about the use of the addition symbol: is it really analogous to the operation, expressed in $(4 + 3) \longrightarrow 7$, as the child perceives it? Perhaps separation of 7 into 4 and 3 would be more adequately expressed by $7 \longrightarrow (4, 3)$ and corresponding actions on objects performed in such a way that $7 \longrightarrow (4, 3)$ would be different from $7 \longrightarrow (3, 4)$.

This investigation suggests that the sentences $(3 + 4) \longrightarrow 7$ and $7 \longrightarrow (3 + 4)$ portray non-symmetric situations, as children perceive them, thus suggesting that the equality symbol, intended to have the symmetric property, is not the most appropriate one to use.

The matter of equality and the basic operations is central to the elementary school mathematics curriculum and beyond. The investigation

[4]Frazer, C. D. "Abilities of College Students to Involve Symmetry of Equality With Applications of Mathematical Generalizations," Florida State University, Tallahassee, Florida, 1976.

described in this paper is only a beginning of the kind of research that should continue. The main goal of the research should be to <u>understand</u> how children, as a result of their early experiences with mathematics, come to formulate mental constructs which possibly dominate their thinking for a long time.